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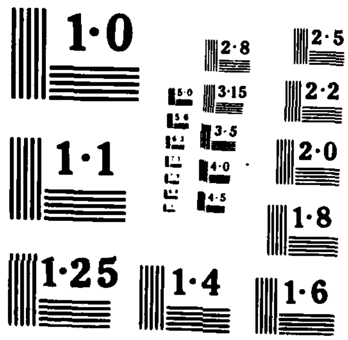
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THREE AND FOUR MODE FACTOR ANALYSIS
WITH APPLICATION TO ASVAB DATA

Shin-ichi Mayekawa

ONR Technical Report 85-7

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THREE AND FOUR MODE FACTOR ANALYSIS WITH APPLICATION TO ASVAB DATA*

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Abstract

The traditional factor analytic view of the PARAFAC model and its extension to a four mode situation with the derivation of the maximum likelihood estimation procedure by the generalized EM algorithm was presented. The four mode model was applied to six data matrices defined by three specialty, (clerical, mechanical, and electrical), times two services, (Air Force and Marine Corps) and successfully recovered the usual four dimensional structure without any rotation. The specialty and service differences was expressed in terms of different weighting of the common factor structure. A model which allows us to compare the factor score means was also investigated.

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Introduction

Factor analysis is a data reduction technique to represent a set of variables in a lower dimensional space. That is, the purpose of factor analyis is to represent a set of variables as a linear combination of a smaller set of latent variables, namely, factors and an overall mean. Given measurements on a set of variables, the analysis results in the estimate of factors, i.e., factor scores, the regession coefficients of each variables on the factors, i.e., factor loadings, and the error variances. When the factors are assumed to be oblique the estimate of the factor correlations is also provided.

When measurements of a set of variables are available from several data sources we could analyze each data matrix independently and compare the results. This approach results in as many sets of factor scores, factor loadings and error variances estimates as the number of data sources. A more parsimonious way is to analyze all the data matrices jointly, with some additional assumptions, reducing the number of parameters to be estimated.

Several methods have been proposed to accomplish this parsimony. For example, if we assume that all the data sources share the same factor loadings we have the factorial invariance model proposed by Lawley and Maxwell(1963) and Meredith(1964a, 1964b). A general estimation procedure under this model is found in Joereskog(1971) where he expresses the degrees of factor invariance in terms of the strength of the equality assumption among parameters. In this approach the main interest is to test whether each factor loadings matrix is the same or not. That is, the equality, or the factor invariance, is treated as an external assumptions to be tested rather than as part of structural model.

Another approach, proposed by Tucker(1966) and later by Harshman(1970), (also Harshman and Lundy(1984)), incorporates some equality of the parameters as an essence of the structural model. For example, Harshman's model, called the PARAFAC model, expresses each data matrix as a product of (common) factor scores, (common) factor loadings, and the weight matrix which represents the differences among the data sources. Tucker's model, called the Three Mode Factor Analysis (TMFA) model, introduces another set of parameters in a core matrix which describes the relationship or the interactions among the variables and the data sources. It is known, Harshman and Lundy(1984), that the PARAFAC model is a special case of Tucker's TMFA model. Several estimation procedures are available for both models. For the PARAFAC model, Harshman(1970, 1972) provides a least squares estimation method and Sands and Young(1980) provide a nonmetric extension of the least squares solution. For the TMFA model, least squares estimation is provided by Krooneberg and de Leeuw(1980) and, maximum likelihood estimation, by Bentler and Lee(1978, 1979).

One advantage of the PARAFAC model is that it provides an unique set of factors in the sense that there are no degrees of freedom for rotation. In contrast, the TMFA model, though it provides a better reproduction of the data because of its generality, does not have this uniqueness property. Naturally, the PARAFAC model is more parsimonious and easier to interpret. Its disadvantage is that is will generally yield larger error variances.

However, the PARAFAC model is not usually expressed in terms of traditional factor analysis terminology and not yet widely used.

The purpose of this paper is to provide a traditional factor analytic view of the PARAFAC model and its maximum likelihood estimation procedure. Also, an extension to the multimode case is discussed.

Model

Suppose that there are p variables, s data sources, N_k, $k=1,2,\ldots,s$, observations from each data source, and r factors. The PARAFAC model can be expressed in terms of factor scores, factor loadings, data source weights and error terms as follows.

$$(1)\quad \underline{y}_{ik} = \underline{m}_k + A\,W_k\,\underline{f}_{ik} + \underline{e}_{ik},$$

$$i=1,2,\ldots,N_k,\ k=1,2,\ldots,s,$$

where

$\underline{y}_{ik}$ is the p x 1 vector of observations on the individual i in the data source k,

$\underline{m}_k$ is the p x 1 grand mean vector for the data source k,

A is the p x r factor loadings matrix common to all the data sources,

W_k is the r x r diagonal weight matrix for the data source k,

$\underline{f}_{ik}$ is the r x 1 factor score vector of individual i in the data source k,

and

$\underline{e}_{ik}$ is the p x 1 error vectors of individual i in the data source k.

Also, the parameters associated with each individual are assumed to have the

following properties.

(2) $\underline{E}[\ \underline{f}_{ik}\ ] = \underline{0},\ i=1,2,\ldots,N_k,\ k=1,2,\ldots,s,$

$\underline{D}[\ \underline{f}_{ik}\ ] = C_F,\ i=1,2,\ldots,N_k,\ k=1,2,\ldots,s,$

$\underline{E}[\ \underline{e}_{ik}\ ] = \underline{0},\ i=1,2,\ldots,N_k,\ k=1,2,\ldots,s,$

and

$\underline{D}[\ \underline{e}_{ik}\ ] = D_k,\ i=1,2,\ldots,N_k,\ k=1,2,\ldots,s,$

where $\underline{E}$ and $\underline{D}$ are, respectively, expectation and dispersion operators and D_k, k=1,2,...,s is assumed to be diagonal.

Also, it is assumed that each $\underline{f}$ and $\underline{e}$ are uncorrelated. To avoid the multiplicative indeterminacy it is assumed that diag[C_F] = I_r and the average of squared W_k matrices is equal to the identity matrix of size r. The additive indetermiancy

(3) $\underline{m}_k + A\ W_k\ \underline{f}_k + A\ W_k\ (\ \underline{f}_{ik} - \underline{f}_k\ )$

or

$$\underline{m}_k + A\ \underline{m} + A\ W_k\ (\ \underline{f}_{ik} - W_k^{-1}\ \underline{m}\ )$$

is taken care of by setting the mean of the factor scores equal to zero.

Usually the PARAFAC model assumes that each individual is measured s times on the same set of variables, (i.e., under s occasions), but here we assume that each data source has different set of individuals. By assuming that each factor score has the same factor dispersion matrix, C_F, the uniqueness property of the original model is retained.

In terms of the dispersion matrix, the model can be written as

(4) $C_k = A\, W_k\, C_F\, W_k\, A' + D_k$, $k=1,2,\ldots,s$,

where C_k is the p x p sample dispersion matrix of the data source k.

The uniqueness property is the result of the fact that after any nonsingular transformation of the factors the model cannot be expressed as the product of the p x r matrix, the r x r diagonal matrix and the r x 1 vector without changing the goodness of fit. That is, the set of s identities

$A\, W_k\, \underline{f}_{ik} = A\, W_k\, T^{-1}\, T\, \underline{f}_{ik}$, $k=1,2,\ldots,s$,

where T is the r x r nonsingular matrix cannot always be expressed as

$B\, V_k\, (\, T\underline{f}_{ik}\,)$, $k=1,2,\ldots,s$,

where V_k, $k=1,2,\ldots,s$ is a diagonal matrix.

The model statated above can be interpreted in various ways. First, by defining a new variable

(5) $\underline{z}_{ik} = W_k \underline{f}_{ik}$, $i=1,2,\ldots,N_k$, $k-1,2,\ldots,s$,

we have

(6) $\underline{y}_{ik} = \underline{m}_k + A\, \underline{z}_{ik} + \underline{e}_{ik}$,

or

(7) $C_k = A\, C_{Zk}\, A' + D_k$,

where $C_{Zk} = \underline{D}[\, \underline{z}_{ik}\,] = W_k\, C_F\, W_k$,

which is a special case of the factorial invariance model where the differences among the data sources are expressed in terms of the differences among the factor dispersions and the error variances. Interestingly, this form of factorial invariance model has not been proposed to this author's knowledge. The reason seems to be that the usual selection theorem does not imply this

form of relationship among the factor correlation matrices.

Another way to interpret the model is as follows. By defining a new factor loadings matrix

(8) $B_k = A\, W_k$, $k=1,2,\ldots,s$,

we have

(9) $\underline{y}_{ik} = \underline{m}_k + B_k\, \underline{f}_{ik} + \underline{e}_{ik}$,

or

(10) $C_k = B_k\, C_F\, B_k' + D_k$,

where the differences are explained in terms of the differences among the factor loadings or the regression coefficients of each variable on the factors.

In either case, the elements of W_k matrix are considered to be the relative weights/importance of the factors. That is, when the (11) element of the W_k matrix has a relatively high value, the implication is that the 1^{th} factor is relatively more important than the rest of the factors in the k^{th} data source. However, since the mean of each factor across the individuals are set equal to zero, this does not mean that those variables whose factor loadings are high on the 1^{th} factor have higher values. Instead, the larger weight generally implies larger variances of those variables whose factor loadings are high on the 1^{th} factor. For example, if the k^{th} data source is highly selective on the basis of those variables whose factor loadings are high on the 1^{th} factor, we should expect that the mean of those variables is high and that the weight of the 1^{th} factor in the k^{th} data source is low, resulting in the high-mean and the small-variance of those variables in the

k^{th} data source.

As mentioned before, the model we are dealing with does not allow us to compare the factor score means of each data source. When it is desired, a slightly different formulation of the model must be used. Severel methods which enables the comparison of the factor score means will be discussed in the later section.

Extention to the Four Mode Situation

Suppose that each data source can be expressed as a combination of two categories. For example, if a test battery is administered to a set of individuals we could divide the entire sample into six subsamples defined by sex (male, female) and race (black, white, other). In this case, it may be more parsimonious to express the variations among the data sources by the product of two weight matices, namely, one associated with sex and anther with race.

Generally, if the s data sources can be regarded as the result of s1 x s2 classification we could write that

(11) $W_k = W1_{k1}\ W2_{k2}$, $k1=1,2,\ldots,s1$, $k2=1,2,\ldots,s2$,

where $k = (k1-1)*s2 + k2$.

The idea is similar to the usual decomposition of the cell means into the combination of the column and row effect in s1 x s2 factorial experimental design where the decompotition is additive rather than multiplicative. General form of this decomposition is known as the Canonical Decomposition of N-Way Tables. (See Carrol and Pruzansky 1984). Those weight matrices can be interpreted similarly as before.

If it is desired to decompose the means we could have the usual ANOVA decomposition such as

(12) $\underline{m}_k = \underline{m} + \underline{m}_{k1} + \underline{m}_{k2}$, k1=1,2,...,s1, k2=1,2,...,s2,

where k = (k1-1)*s2 + k2.

The Maximum Likelihood Estimation by the EM Algorithm

With the additional assumption of normality

(13) $\underline{f}_{ik} : N_r(\ \underline{0},\ C_F\ )$, i=1,2,...,$N_k$,k=1,2,...,s, iid,

and

$\underline{e}_{ik} : N_p(\ \underline{0},\ D_k\ )$, i=1,2,...,$N_k$, iid, k=1,2,...,s,

and the statistical independence of the factor scores and the error terms, we have

(14) $S_k : W_p(\ Q_k,\ N_k\ )$, k=1,2,...,s,

where S_k is the sample mean corrected SSCP matrix,

$$Q_k = A\ W_k\ C_F\ W_k\ A' + D_k,$$

and

$W_p(\ A,\ df\ )$ denotes the p-variate Wishart distribution with the degrees of freedom df and the mean df x A.

Here the parameter $\underline{m}_k$, k=1,2,...,s, is estimated by the sample mean and treated as a constant when deriving the Wishart distributions.

The MLE based on this model can be found by differenciating the product of s Wishart likelihood functions with respect to A, C_F, D_k, and W_k, k=1,2,...,s. However, noticing that the Wishart likelihood presented above is

the marginal likelihood of $\underline{f}_{ik}$, A, C_F, W_k, D_k, i=1,2,...,N_k, k=1,2,...,s, with all the $\underline{f}_{ik}$'s integrated out, we could instead use the following EM algorithm where the factor scores are treated as missing data. This approach, originally advocated by Rubin and Thayer (1982) in the standard factor analysis context, has the definite advantage of simplicity of the calculation involved due to the linear (tri-linear) nature of the complete data likelihood.

The application of the General EM algorithm scheme in this context is outlined below. For further discussion of the EM algorithm see Mayekawa(1985). In the E-step, the expectation of the log complete data likelihood with respect to the conditional distribution of the factor scores given data, factor loadings, data source weights, and the error variances is calculated. The complete data likelihood is given by

(15) $L = f(Y \mid F, A, W, D) \times f(F \mid C_F)$,

where

(16) $f(Y \mid F, A, W, D) =$

$$\prod_{k=1}^{s} [\, f(Y_k \mid F_k, A, W_k, D_k) \,],$$

where

(17) $f(Y_k \mid F_k, A, W_k, D_k) =$

$$\prod_{i=1}^{Nk} [\, f(\underline{y}_{ik} \mid \underline{f}_{ik}, A, W_k, D_k) \,],$$

where

(18) $-2 \ln f(\underline{y}_{ik} \mid \underline{f}_{ik}, A, W_k, D_k) =$

$$(\underline{y}_{ik} - AW_k\underline{f}_{ik})'D_k^{-1}(\underline{y}_{ik} - AW_k\underline{f}_{ik})$$

$+ \ln |D_k| +$ constant which does not involve the parameters,

and

(19) $f(F \mid C_F) = \prod_{k=1}^{s} [f(F_k \mid C_F)]$,

where

(20) $-2 \ln f(F_k \mid C_F) =$

$\sum_{i=1}^{N_k} [(\underline{f}_{ik}' C_F^{-1} \underline{f}_{ik})]$

$+ N_k \times \ln |C_F|$

$+$ constant.

It should be noted that, in order to avoid notational complexity, the mean deviation score, $\underline{y}_{ik} - \underline{m}_k$, is denoted by $\underline{y}_{ik}$ in the above expressions and throughout this section. Also, the $N_k \times p$ column centered data matrix of the data source k is denoted by Y_k, and the $N_k \times r$ factor score matrix, by F_k.

The conditional distribution of the factor score is

(21) $\underline{f}_{ik} \mid Y, A, W, D, C_F \;:\; N_r(\underline{f}_{ik}^{*}, V_k^{*})$,

$i = 1,2,\ldots, N_k$, i.i.d, $k=1,2,\ldots,s$,

where

(22) $V_k^{*} = (W_k A' D_k^{-1} A W_k + C_F^{-1})^{-1}$, $k=1,2,\ldots,s$,

and

(23) $F_k^{*} = Y_k \; D_k^{-1} \; A \; W_k \; V_k$, $k=1,2,\ldots,s$,

and the expectation of ln L is given by substituting $\underline{f}_{ik}^{*}$ for $\underline{f}_{ik}$ in (18)

and adding a term

(24) $\mathrm{tr}\ W_k A' D_k^{-1} A W_k\ V_k^*$

and

$$N_k\ \mathrm{tr}\ C_F^{-1}\ V_k^*$$

to (18) and (20), respectively. The result can be expressed as

(25) $\underline{E}[\ \ln L\ ] = \sum_{k=1}^{s} [$

$\mathrm{tr}[\ (\ Y_k - F_k^* W_k A'\ )\ D_k^{-1}\ (\ Y_k - F_k^* W_k A'\ )'\ ]$

$+ N_k\ \ln |D_k|$

$+ N_k\ \mathrm{tr}\ W_k A' D_k^{-1} A W_k V_k^*$

$+ \mathrm{tr}\ C_F^{-1}\ F_k^{*\prime} F_k^* \quad + \quad N_k\ \mathrm{tr}\ C_F^{-1}\ V_k^*$

$+ N_k\ \ln |C_F|\ \ ]$

+ constant which doe t involve the parameters.

Since the conditional expectation can be expressed as a function of $F_k^{*\prime} F_k^*$ and $Y_k' F_k^*$, the N_k x r matrices, F_k^*'s, need not be stored in the course of calculation. Thus, the E-step can be summarized as

The E-step.

(26) $V_k^* = (\ W_k A' D_k^{-1} A W_k + C_F^{-1}\ )^{-1}$, k=1,2,...,s,

(27) $Y_k' F_k^* = S_k\ D_k^{-1}\ A\ W_k\ V_k^*$, k=1,2,...,s,

and

(28) $F_k^{*\prime} F_k^* = V_k^*\ W_k\ A'\ D_k^{-1}\ Y_k' F_k^*$, k=1,2,...,s.

In the M-step the conditional expectation of the log complete data likelihood is maximized with respect to A, W_k, D_k, , k=1,2,...,s, and C_F treating F_k^* and V_k^* as constant.

The M-step.

(29) $\underline{a}_j = \sum_{k=1}^{s}[\ (W_k(F_k^{*\prime}F_k^*+N_kV_k^*)W_k/d_{jk})^{-1}\ ]$

$\quad x \sum_{k=1}^{s}[\ \ W_k\ F_k^{*\prime}\underline{y}_{jk}\ /\ d_{jk}\ ]$, j=1,2,...,p,

(30) $d_{jk} = (\ RSS_{jk} + N_k\ \underline{a}_j{}'W_kV_k^*W_k\underline{a}_j\ )\ /\ N_k$,

j=1,2,...,p, k=1,2,...,s,

where $RSS_{jk} = (\ \underline{y}_{jk}-F_k^*\underline{a}_j\ )'(\ \underline{y}_{jk}-F_k^*\underline{a}_j\ )$,

(31) W_k = diag[$\underline{c}$], k=1,2,...,s,

where $\underline{c} = T^{-1}\ \underline{h}$,

$h_l = \sum_{j=1}^{p}[\ a_{jl}/d_{jk}\ (Y_k{}'F_k^*)_{jl}\ ]$, l=1,2,...,r,

$t_{lm} = \sum_{j=1}^{p}[\ a_{jl}a_{jm}/d_{jk}\ ]$

$\quad x\ (F_k^{*\prime}F_k^* + N_k\ V_k^*)_{lm}$, l,m=1,2,...,r,

and

(32) $C_F = (1/N_+) \sum_{k=1}^{s}[\ F_k^{*\prime}F_k^* + N_k\ V_k^*\ ]$,

where $N_+ = \sum_{k=1}^{s}[\ N_k\ ]$.

The above formuli are derived by taking the partial derivative of the

conditional expectation of the complete log likelihood with respect to each parameters and solving the resulting normal equations independently of the normal equations for the rest of the parameters. Therefore, strictly speaking, they do not provide ths maximum of the conditional expectation but merely increase the value of the conditional expectation.

In order to avoid multiplicative redundancy, the diagonal elements of the C_F matrix should be normalized to the identity matrix and the W_k matrix should be also normalized so that the average of the squared W_k matrices is the identity matrix.

As in standard factor analysis, the mean vectors and the SSCP matrices of each data source are sufficient to estimate the parameters A, W_k,k=1,2,...,s, and D_k, k=1,2,...,s.

Optionally, we could enforce some equality restrictions such as

(33) $D_1 = D_2 =, \ldots, D_s = D,$

(34) $D_k = d_k I_p$, k=1,2,...,s,

or

(35) $D_1 = D_2 =, \ldots, D_s = d.$

With the last restriction the MLE is equivalent to the least squares estimates. Also, when the factors are assumed to be orthogonal we simply skip the estimation of C_F holding it to the identity matrix.

The M-step for the W1 and W2 matrices can be derived using a similar linearization technique. That is, noticing that the conditional distribution of $\underline{f}_{ik}$ and the conditional expectation of ln L is the same as (21), (22),

(23) and (25) with W_k matrix and the subscript k defined by (11), all the E-step and the M-step except for (31) remain the same. For the data source weight matrix, (31) should be modified as follows:

(36) $W1_{k1} = \mathrm{diag}[\ \underline{c}\ ]$, $k1=1,2,\ldots,s1$,

where $\underline{c} = \left(\sum_{k2=1}^{s2} [\ T_{k2}\ ] \right)^{-1} \sum_{k2=1}^{s2} [\ \underline{h}_{k2}\ ]$,

$$h_{k2\ 1} = \sum_{j=1}^{p} [a_{j1} w2_{k2\ 11} / d_{jk} (Y_{k1\ k2}{}' F^{*}_{k1\ k2})_{j1}]$$

$1=1,2,\ldots,r$,

$$t_{k2\ 1m} = \sum_{j=1}^{p} [\ a_{j1} a_{jm} w2_{k2\ 11} w2_{k2\ mm} / d_{jk}\ ]$$

$$\times\ (F^{*}_{k1\ k2}{}' F^{*}_{k1\ k2} + N_{k1\ k2}\ V^{*}_{k1\ k2})_{1m},$$

$1,m = 1,2,\ldots,r$,

and

(37) $W2_{k2} = \mathrm{diag}[\ \underline{c}\ ]$, $k2=1,2,\ldots,s2$,

where $\underline{c} = \left(\sum_{k1=1}^{s1} [\ T_{k1}\ ] \right)^{-1} \sum_{k1=1}^{s1} [\ \underline{h}_{k1}\ ]$,

$$h_{k1\ 1} = \sum_{j=1}^{p} [a_{j1} w1_{k1\ 11} / d_{jk} (Y_{k1\ k2}{}' F^{*}_{k1\ k2})_{j1}]$$

$1=1,2,\ldots,r$,

$$t_{k1\ 1m} = \sum_{j=1}^{p} [\ a_{j1} a_{jm} w1_{k1\ 11} w1_{k1\ mm} / d_{jk}\ ]$$

$$\times\ (F^{*}_{k1\ k2}{}' F^{*}_{k1\ k2} + N_{k1\ k2}\ V^{*}_{k1\ k2})_{1m},$$

$1,m = 1,2,\ldots,r$.

Also, normalizations such as setting the average squared W_k matrices and the average squared $W1_{k1}$ matrices to the identity matrix shall be enforced.

Initial Configuration

The most efficient way to calculate the initial configuration seems to be the application of the SUMSCAL algorithm advocated by de Leeuw and Pruzansky(1978). The method, which assumes orthogonality of the factors and is restricted to the three mode situation, has been used in Novick, et. al. (1983). When a four or higher mode model is used, the log additive decomposition of the initial W_k matrices should provide a reasonable estimate of each weight matrix.

Standardization of the Raw Data

Since the MLE has a nice property of scale invariance we may be able to rescale each variable to a desired form. Usual practice in standard factor analysis is to scale each variable so that each has zero mean and unit variance. However, as pointed out by Joereskog(1971) and Harshman and Lundy(1984), standardization within each data source changes the form of the likelihood. That is, the rescaling must be performed, after subtracting each within data source mean, by multiplying a common constant across all the data sources to each variable. The most convenient approach is to rescale the variables so that the average of the rescaled dispersion matrix has unit diagonal elements. The number of individuals may or may not be used to weigh the averaging process.

Analysis of ASVAB Data Set

The method proposed in the previous sections is applied to a subset of ASVAB Form 8.

The variables analyzed are:

1. General Science (GS)
2. Arithmetic Reasoning (AR)
3. Word Knowledge (WK)
4. Paragraph Comprehension (PC)
5. Numerical Operations (speeded) (NO)
6. Coding Speed (speeded) (CS)
7. Auto-Shop Information (AS)
8. Mathematics Knowledge (MK)
9. Mechanical Comprehension (MC)
10. Electronics Information (EI)

The means and the standard deviations are shown in Table 1. This set of variables are known to have the following four factors, see, for example, Ree, Mullins, Mathews and Massey(1981):

1. Verbal: variables 1, 3, 4
2. Technical: variables 7, 9, 10
3. Mathematical: variables 2, 8
4. Speeded: variables 5, 6

It is also known that these factors are positively correlated, with most correlation in the .4 range.

The scores of these ten variables are available for the following six data sources which are the combinations of two different armed services, (Marine

Corps, Air Force) and three different specialties, (Clerical, Mechanical, Electrical). The number of individuals in each data source are:

1. MC - CLE 3285
2. MC - MEC 3118
3. MC - ELE 1415
4. AF - CLE 8963
5. AF - MEC 16884
6. AF - ELE 7897

In the analysis we assume only that there are four factors and attempt to demonstrate that the usually accepted pattern of factor loadings can be found using the PARAFAC model.

The six sample dispersion matrices are first rescaled so that the diagonal elements of the weighted average are equal to unity. The resulting rescaled dispersion matrices are shown in Table 2.

The four mode analysis of this data set by the maximum likelihood method proposed in the previous sections, with $r = 4$, resulted in the parameter estimates shown in Table 3. The error variances are assumed to be equal across the data sources. The diagonal elements of each W matrix are arranged to form data source x factor matrix in Table 3.

First, it should be noted that, without any rotation, the four mode analysis recovered those four dimensions found by the standard two mode analysis. According to Ree, et.al.(1981), we could name the first factor Technical, the second, Speeded, the third, Verbal, and the fourth, Mathematical. The major difference is in the factor dispersion matrix: their solutions is more oblique whereas the highest factor correlation in our

solution is about 0.2. As a result, the Technical factor, first factor, is more influential than their corresponding factor. Once again, we emphasi e that NO rotations are performed on the final result.

Second, the inspection of the W1 matrices confirms the fact that the Air Force is more selective in general. This is shown by the smaller value of the W1 weight matrix which represents the difference between the Air Force and the Marine Corps. In particular, the Air Force is highly selective on the Speeded Factor, second factor. The means of variables 5 and 6, which are highly loaded on the factor, in Table 1 shows that in all three specialty areas their means are higher than those of the Marine Corps.

Also, the W2 matrix shows that the mechanical specialty and the clerical specialty area has a smaller weight on, respectively, Technical and Speeded factors. This, combined with the inspections of the means and the standard deviations in Table 1, shows that mechanical specialists are homogeneous in those variables which are highly loaded on the Technical factor and also have higher scores on those variables. The same argument should be applied to the clerical specialist with respect to the Speeded factors. As the result, the Air Force - Clerical specialtist has the smallest variances of the variables 5 and 6, which can be confirmed by Table 1 and Table 2.

Discussion

In this section we discuss a slightly different formulation of the model which enables us to compare the factor score means. Consider the additive indeterminacy in (3). The formula says that the subtraction of the mean factor score, $\underline{f}_k$, can be compensated by the addition of corresponding quantity to

$\underline{m}_k$. In the previous section we removed this redundancy by setting the factor score mean equal to zero. This approach is equivalent to defining $\underline{m}_k$ as the mean of the observation,

$$(38)\quad \underline{E}[\ \underline{y}_{ik}\ ] = \underline{m}_k + A\ W_k\ \underline{f}_k$$
$$= \underline{m}_k.$$

The reason why we chose to use this method is that this is the usual constraint in standard factor analysis where the factor score mean is not of interest. There are, however, other ways to remove this redundancy, especially, in the PARAFAC situation. For example, we could set all the $\underline{m}_k$'s equal to zero,

$$(39)\quad \underline{E}[\ \underline{y}_{ik}\ ] = A\ W_k\ \underline{f}_k,$$

and treat $\underline{f}_k$'s as additional parameters to be estimated. The implication of this formulation is that, within each data source, if a subset of variables has the similar factor loadings they must have similar means. That is, if variable j and j' have identical factor loadings, i.e., if the j^{th} and the j'^{th} row of the A matrix are identical, their mean must be identical within each data source. This may not be a realistic assumption in practice. For example, when a test and its half-test is analyzed together we expect that the means of the half-test is about half of the mean of the full-test while expecting that the both tests have similar factor loadings. Another way to reduce the redundancy is to assume

$$(40)\quad \underline{E}[\ \underline{y}_{jk}\ ] = \underline{m} + A\ W_k\ \underline{f}_k.$$

Since there are some redundancies left in (40) we further define $\underline{m}$ as the grand mean across all the data sources. (The restriction that the average of $\underline{f}_k$ is

equal to $\underline{0}$ can also remove the remaining redundancy.) Note that this is more restrictive than (39) since (39) does not enforce any structural restrictions on each mean while (40) assumes that each mean is a sum of the grand mean and a vector which lies in the column space of $A\ W_k$. Therefore, we may say that this approach is a compromize between our original approach, (38), and the more restrictive case (39).

The analysis under this assumption may be perfomed by modifying the conditional distribution of $\underline{f}_{ik}$ in (21) and resulting expectation of the log complete data likelihood. The factor score means should be estimated in the M-step.

Summary

The traditional factor analytic view of the PARAFAC model and its extension to a four mode situation with the derivation of the maximum likelihood estimation procedure by the generelized EM algorithm was presented. The four mode model was applied to six data matrices defined by three specialty, (clerical, mechanical, and electrical), times two services, (Air Force and Marine Corps) and successfully recovered the usual four dimensional structure without any rotation. The specialty and service differences was expressed in terms of different weighting of the common factor structure. A model which allows us to compare the factor score means was also investigated.

References

Bentler, P. M. and Lee, S. Y. (1978). Statistical aspect of a three-mode factor analysis model. Psychometrika, 43, 344-352.

Bentler, P. M. and Lee, S. Y. (1979). A statistical development of three-model factor analysis. British Journal of Mathematical and Statistical Psychology. 32, 87-104.

Carrol, J. D., and Pruzansky, S. (1984). The CANDECOMP-CANDELINK family of model and methods for multidimensional data analysis. In Law, H. G., Snyder, C. W., Hattie, J. A., and McDonald, R. P. (Eds.), Research methods for multimode data analysis. New York: Praeger, 122-215.

Harshman, R. A. (1970). Foundations of PARAFAC procedure: Models and conditions for an 'explanatory' multi-modal factor analysis. UCLA working papers in Phonetics, 16, 1-84. (University Microfilms No. 10,085).

Harshman, R. A. (1972). PARAFAC2: Mathematical and technical notes. UCLA working papers in Phonetics, 22, 31-44. (University Microfilms No. 10,085).

Harshman, R. A. and Lundy, M. E. (1984). The PARAFAC model for three-way factor analysis and multidimensional scaling. In Law, H. G., Snyder, C. W., Hattie, J. A., and McDonald, R. P. (Eds.), Research methods for multimode data analysis. New York: Praeger, 122-215.

de Leeuv J. and Pruzansky, S. (1978). A new computational method to fit the weighted Euclidean distance model. Psychometrika, 43, 479-490.

Joereskog, K. G. (1971). Simultaneous factor analysis in several populations. Psychometrika, 36, 409-426.

Kooneberg, P. M. and de Leeuv, J. (1980). Principal component analysis of three-mode data by means of alternating least squares algorithm. Psychometrika, 45, 69-97.

Lawley, D. N. and Maxwell, A. E. (1963). Factor analysis as a statistical method. London: Butt rworths. 93-101.

Mayekawa, S. Bayesian factor analysis. (ONR Technical Report 85-3). Iowa City, IA: The University of Iowa, College of Education.

Meredith, W. (1964a). Rotation to achieve factorial invariance. Psychometrika, 19, 187-206.

Meredith, W. (1964b). Notes on factorial invariance. Psychometrika, 19, 177-185.

Novick, M. R., Chen, J. J., Mayekawa, S., Woodworth, G. G., Cook, J., and Novick, R. L. (1983). Manual for the Computer Assisted Data Analysis (CADA) Monitor (1983). Iowa City, IA: Authors.

Ree, M. J., Mullins, C. J., Mathews, J.J., and Massey, R. H. Armed services vocational aptitide battery: item and factor analysis of forms 8, 9, and 10. AFHRL-TR-81-55, Manpower and personnel division, Brooks, Air Force Base, Texas.

Rubin, D. B. and Thayer, D. T. (1982). EM Algorithm for ML Factor Analysis. Psychometrika, 47, 69-76.

Sands, R., and Young, F. W. (1980). Component model for three-way data: an alternating least squares algorithm with optimal scaling features. Psychometrika, 45, 39-67.

Tucker, L. R. (1966). Some mathematical notes on three-mode factor analysis. Psychometrika, 31, 279-311.

Table 1. Means and Standard Deviations of the Original Variables

Means

	MC-CLE	MC-MEC	MC-ELE	AF-CLE	AF-MEC	AF-ELE
GS	16.940	17.030	17.850	15.986	17.332	18.507
AR	20.490	19.110	20.260	19.755	19.715	22.248
WK	28.140	26.640	27.900	27.436	27.175	28.624
PC	11.490	10.870	11.120	11.789	11.550	12.131
NO	41.170	37.140	37.590	43.316	38.167	39.968
CS	55.070	47.200	47.700	55.475	47.160	49.460
AS	15.970	18.730	17.030	14.307	18.921	18.670
MK	14.810	13.280	15.100	14.291	13.509	16.256
MC	15.800	17.390	16.510	14.238	17.244	17.954
EI	12.450	13.650	13.630	11.599	13.535	14.394

Standard deviations

	MC-CLE	MC-MEC	MC-ELE	AF-CLE	AF-MEC	AF-ELE
GS	3.832	3.675	3.683	3.684	3.335	3.538
AR	5.609	5.106	5.441	5.114	4.981	5.169
WK	4.888	4.880	4.863	4.545	4.569	4.628
PC	2.503	2.582	2.766	1.951	2.098	2.050
NO	8.090	8.800	9.010	5.779	7.483	7.305
CS	15.021	14.494	15.008	10.925	11.648	12.262
AS	4.928	4.127	4.700	4.623	3.737	4.280
MK	4.944	4.529	4.859	4.537	4.428	4.971
MC	4.643	4.046	4.533	4.257	3.732	3.937
EI	3.470	3.005	3.213	3.178	2.869	3.021

Number of observations

MC-CLE	MC-MEC	MC-ELE	AF-CLE	AF-MEC	AF-ELE
3258	3118	1415	8963	16884	7897

Table 2: Sample Dispersion Matrices

MC - CLE

	1	2	3	4	5	6
1	1.178636	0.517880	0.746108	0.579417	0.119386	0.079585
2	0.517880	1.198937	0.505769	0.571901	0.471982	0.370015
3	0.746108	0.505769	1.112072	0.708778	0.164813	0.124974
4	0.579417	0.571901	0.708778	1.343676	0.310126	0.300254
5	0.119386	0.471982	0.164813	0.310126	1.215994	0.701767
6	0.079585	0.370015	0.124974	0.300254	0.701767	1.497889
7	0.667442	0.492772	0.429781	0.426459	0.151502	0.153605
8	0.484964	0.789714	0.437370	0.472698	0.406709	0.308891
9	0.652177	0.644871	0.470192	0.514137	0.227436	0.273499
10	0.705888	0.475099	0.529311	0.466305	0.134747	0.124527

	7	8	9	10
1	0.667442	0.484964	0.652177	0.705888
2	0.492772	0.789714	0.644871	0.475099
3	0.429781	0.437370	0.470192	0.529311
4	0.426459	0.472698	0.514137	0.466305
5	0.151502	0.406709	0.227436	0.134747
6	0.153605	0.308891	0.273499	0.124527
7	1.371366	0.342356	0.868547	0.885173
8	0.342356	1.143990	0.548740	0.414558
9	0.868547	0.548740	1.335946	0.795607
10	0.885173	0.414558	0.795607	1.303493

Table 2 (continued)

MC - MEC

	1	2	3	4	5	6
1	1.083785	0.339531	0.673119	0.552077	0.062664	0.029272
2	0.339531	0.993451	0.313733	0.406963	0.373296	0.270903
3	0.673119	0.313733	1.108441	0.666283	0.115560	0.110871
4	0.552077	0.406963	0.666283	1.430571	0.313665	0.276246
5	0.062664	0.373296	0.115560	0.313665	1.438242	0.721064
6	0.029272	0.270903	0.110871	0.276246	0.721064	1.394635
7	0.418566	0.231351	0.310545	0.289679	−0.017714	0.014750
8	0.382086	0.602129	0.350146	0.392206	0.330758	0.228661
9	0.434441	0.386068	0.309557	0.395292	0.066320	0.141343
10	0.494307	0.230606	0.394469	0.375685	−0.003811	0.042069

	7	8	9	10
1	0.418566	0.382086	0.434441	0.494307
2	0.231351	0.602129	0.386068	0.230606
3	0.310545	0.350146	0.309557	0.394469
4	0.289679	0.392206	0.395292	0.375685
5	−0.017714	0.330758	0.066320	−0.003811
6	0.014750	0.228661	0.141343	0.042069
7	0.961952	0.146613	0.533945	0.537735
8	0.146613	0.959664	0.333154	0.218805
9	0.533945	0.333154	1.014227	0.491059
10	0.537735	0.218805	0.491059	0.977577

Table 2 (continued)

MC - ELE

	1	2	3	4	5	6
1	1.088970	0.444932	0.686065	0.617761	0.175045	0.151668
2	0.444932	1.128117	0.462095	0.590886	0.512778	0.433683
3	0.686065	0.462095	1.100700	0.807166	0.220489	0.225016
4	0.617761	0.590886	0.807166	1.641387	0.388635	0.456438
5	0.175045	0.512778	0.220489	0.388635	1.508016	0.766121
6	0.151668	0.433683	0.225016	0.456438	0.766121	1.495214
7	0.574712	0.458135	0.447760	0.508952	0.207431	0.202605
8	0.465069	0.723645	0.467525	0.546951	0.441771	0.367036
9	0.574713	0.601606	0.474091	0.593750	0.293606	0.314584
10	0.604843	0.410017	0.508383	0.476367	0.229347	0.138149

	7	8	9	10
1	0.574712	0.465069	0.574713	0.604843
2	0.458135	0.723645	0.601606	0.410017
3	0.447760	0.467525	0.474091	0.508383
4	0.508952	0.546951	0.593750	0.476367
5	0.207431	0.441771	0.293606	0.229347
6	0.202605	0.367036	0.314584	0.138149
7	1.247264	0.323559	0.777027	0.684396
8	0.323559	1.105020	0.542661	0.375034
9	0.777027	0.542661	1.272915	0.694490
10	0.684396	0.375034	0.694490	1.117525

Table 2 (continued)

AF - CLE

	1	2	3	4	5	6
1	1.089612	0.374383	0.616486	0.386810	-0.047248	-0.050883
2	0.374383	0.996668	0.264395	0.259636	0.162585	0.101066
3	0.616486	0.264395	0.961422	0.430905	-0.082324	-0.056227
4	0.386810	0.259636	0.430905	0.816532	-0.021440	0.008320
5	-0.047248	0.162585	-0.082324	-0.021440	0.620489	0.272165
6	-0.050883	0.101066	-0.056227	0.008320	0.272165	0.792366
7	0.545702	0.399244	0.367770	0.276713	-0.043238	-0.058872
8	0.361148	0.613614	0.251703	0.238704	0.170579	0.093392
9	0.534856	0.489634	0.351526	0.284509	-0.016586	-0.013364
10	0.550750	0.332561	0.404871	0.263472	-0.050534	-0.057021

	7	8	9	10
1	0.545702	0.361148	0.534856	0.550750
2	0.399244	0.613614	0.489634	0.332561
3	0.367770	0.251703	0.351526	0.404871
4	0.276713	0.238704	0.284509	0.263472
5	-0.043238	0.170579	-0.016586	-0.050534
6	-0.058872	0.093392	-0.013364	-0.057021
7	1.206954	0.265190	0.670281	0.685928
8	0.265190	0.963079	0.436316	0.288179
9	0.670281	0.436316	1.122904	0.576577
10	0.685928	0.288179	0.576577	1.093693

Table 2 (continued)

AF - MEC

	1	2	3	4	5	6
1	0.892798	0.255790	0.538381	0.352900	-0.010703	0.000065
2	0.255790	0.945362	0.256308	0.278090	0.294504	0.177461
3	0.538381	0.256308	0.971718	0.468687	0.001412	0.054271
4	0.352900	0.278090	0.468687	0.943868	0.097563	0.125340
5	-0.010703	0.294504	0.001412	0.097563	1.040331	0.471375
6	0.000065	0.177461	0.054271	0.125340	0.471375	0.900619
7	0.245481	0.185494	0.167392	0.129370	-0.082255	-0.054124
8	0.282560	0.556848	0.261338	0.270063	0.307642	0.185536
9	0.316671	0.357519	0.242488	0.223441	-0.003437	0.033758
10	0.376656	0.211314	0.314900	0.215375	-0.067750	-0.041885

	7	8	9	10
1	0.245481	0.282560	0.316671	0.376656
2	0.185494	0.556848	0.357519	0.211314
3	0.167392	0.261338	0.242488	0.314900
4	0.129370	0.270063	0.223441	0.215375
5	-0.082255	0.307642	-0.003437	-0.067750
6	-0.054124	0.185536	0.033758	-0.041885
7	0.788617	0.081958	0.363429	0.435066
8	0.081958	0.917385	0.306355	0.184430
9	0.363429	0.306355	0.862953	0.383867
10	0.435066	0.184430	0.383867	0.891428

Table 2 (continued)

AF - ELE

	1	2	3	4	5	6
1	1.004770	0.429246	0.644426	0.440293	0.073658	0.085264
2	0.429246	1.018155	0.412641	0.395814	0.352546	0.289226
3	0.644426	0.412641	0.997157	0.522813	0.081628	0.143281
4	0.440293	0.395814	0.522813	0.901530	0.139728	0.179852
5	0.073658	0.352546	0.081628	0.139728	0.991341	0.531685
6	0.085264	0.289226	0.143281	0.179852	0.531685	0.998183
7	0.382372	0.248730	0.268558	0.210098	-0.078564	-0.057613
8	0.488072	0.751722	0.432582	0.383563	0.386840	0.307277
9	0.465788	0.431552	0.347703	0.296424	0.025306	0.059673
10	0.497701	0.332582	0.397299	0.281620	-0.018622	-0.009183

	7	8	9	10
1	0.382372	0.488072	0.465788	0.497701
2	0.248730	0.751722	0.431552	0.332582
3	0.268558	0.432582	0.347703	0.397299
4	0.210098	0.383563	0.296424	0.281620
5	-0.078564	0.386840	0.025306	-0.018622
6	-0.057613	0.307277	0.059673	-0.009183
7	1.034557	0.134562	0.548569	0.560425
8	0.134562	1.156241	0.405702	0.323375
9	0.548569	0.405702	0.960397	0.515497
10	0.560425	0.323375	0.515497	0.988374

Table 3: Parameter Estimates

A-MATRIX: Factor loadings matrix

	1	2	3	4
1	0.559643	0.184060	-0.440180	-0.208001
2	0.475841	0.257108	0.103408	-0.492387
3	0.355282	0.305298	-0.721310	-0.283648
4	0.323135	0.332383	-0.371852	-0.235153
5	-0.008898	0.791521	0.233699	-0.141539
6	-0.000931	0.690510	0.120493	-0.060708
7	0.859566	0.155168	-0.063008	0.276114
8	0.377196	0.140648	0.129718	-0.715394
9	0.760223	0.126622	-0.019828	-0.069300
10	0.753870	0.138062	-0.184281	0.080616

W1-MATRIX: Weight matrix associated with each armed services

	1	2	3	4
1	1.054964	1.191168	1.016724	1.021623
2	0.941834	0.762311	0.982992	0.977899

W2-MATRIX: Weight matrix associated with each specialties

	1	2	3	4
1	1.115465	0.797112	1.013535	0.943578
2	0.857606	1.068602	1.035998	0.960787
3	1.010074	1.105759	0.948396	1.089288

W-MATRIX: Product of W1 and W2 matrices

	1	2	3	4
1	1.176776	0.949494	1.030485	0.963982
2	0.904743	1.272884	1.053324	0.981562
3	1.065592	1.317145	0.964256	1.112842
4	1.050583	0.607648	0.996296	0.922724
5	0.807722	0.814607	1.018378	0.939553
6	0.951322	0.842933	0.932265	1.065213

Table 3 (continued)

D-MATRIX: Error variances

1	0.442958
2	0.422579
3	0.218548
4	0.625186
5	0.394725
6	0.604116
7	0.346037
8	0.260786
9	0.470492
10	0.476524

CF-MATRIX: Factor dispersion matrix

	1	2	3	4
1	1.000000	-0.102711	-0.023244	-0.136183
2	-0.102711	1.000000	0.110366	-0.214670
3	-0.023244	0.110366	1.000000	-0.011888
4	-0.136183	-0.214670	-0.011888	1.000000

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